Date 16-12-20

Ring with unity — If in a ring

R there exists an element denoted by 1 such that $1 \cdot a = a = a \cdot 1$ $\forall a \in R$, then R is called a ring with unit element. The element $1 \in R$ is called the unit element of the ring.

i.e. Obviously 1 is the multiplicative identity of R. Thus if a ring possesses multiplicative identity then it is a ring with unity.

Commutative Ring — If in a Ring

R, the multiplication composition is also commutative. ie, if we have $a \cdot b = b a \;\forall\; a, b \in R$ then R is called a commutative ring.

Elementary properties of a Ring

Theorem If R is a ring then for all $a, b, c \in R$.

(i) $a0 = 0a = 0$.

Proof we have

$a0 = a(0+0)$ $\quad$ ($\because 0+0=0$

$= a0 + a\cdot 0$ $\quad$ by left distributive law.

$\therefore 0 + a0 = a0 + a0$ $\quad$ ($\because a0 \in R$ & $0 + a0 = a0$)

Now R is a group with respect to Addition, therefore applying right cancellation law for addition in R we get $0 = a0$

Similarly we have $0a = (0+0)a$

$= 0a + 0a$

$\therefore 0 + 0a = 0a + 0a$ ($\because 0 + 0a = 0a$)

Applying right cancellation law for addition in R we get

$0 = 0a.$